Elastic Parameter Examples

Rigidity.
The rigidity of quartz is about $6 \times 10^9$ Pa. To produce a shear strain of .01, which corresponds to rotating a line drawn into a block of quartz by about .5 degree, how much shear stress do we need? If we wanted to shear a 1 meter cubed quartz block by gluing it to a wall and then hanging enough weight off the side to produce the .01 shear strain, how much weight would we need?

\[
\sigma_{xy} = 2\mu \varepsilon_{xy}
\]

\[
= 2 \times 6 \times 10^9 \text{ Pa} \times (.01)
\]

\[
= .12 \times 10^9 \text{ Pa.}
\]

\[
= 1.2 \times 10^8 \text{ Pa.}
\]

What's this in more intuitive units? 1 bar of pressure = $10^5$ Pa = about 15 Pounds per square inch, or PSI.

\[
\sigma_{xy} = 1.2 \times 10^3 \text{ bar} = 1200 \text{ atmospheres of pressure, or}
\]

\[
= 18,000 \text{ PSI.}
\]

To produce that much shear strain by hanging something off the side of the block, we need a force equal to

Force = $\sigma_{xy} \times 1 \text{ m}^2 = 1.2 \times 10^8 \text{ Newtons} = \text{Mass} \times \text{Gravity} \Rightarrow$

Mass = $1.2 \times 10^8 \text{ Newtons} / 9.8 \text{ M/S}^2$

\[
= 1.2 \times 10^7 \text{ Kg} = 12 \text{ million Kg or about 25 million pounds.}
\]

A large military transport ship displaces about 12000 tons of water, or about 25 million pounds,

so to produce .01 shear strain, you'd have to hang a Navy cruiser off the side of your quartz cube.

Bulk Modulus

Suppose we want to reduce the volume of a sphere of quartz by 10% by putting a bunch of pressure on it, for example, by immersing it in the bottom of some deep ocean. How much pressure do we need to put on the sphere to achieve this volume reduction? A 10% reduction in volume means reducing a 1 meter radius sphere to 96 cm. For quartz, $k = 1.7 \times 10^9 \text{ PA}$.

We have

\[
\Delta V = (\text{final volume} - \text{original volume})/ \text{original volume} = .9 - 1/1 = -.1
\]
and
\[ k = -\frac{\text{Pressure}}{\Delta V} \Rightarrow \]
Pressure \( = -k \cdot \Delta V \)
\[ = \cdot1 \times 1.7 \text{ GPA} \]
In other units,
\[ = 1.7 \times 10^4 \text{ bar}, \text{ since } 1 \text{ bar} = 10^5 \text{ Pa}, \]
\[ = 255,000 \text{ PSI}, \text{ since } 1 \text{ bar} = \text{ about } 15 \text{ PSI.} \]

The pressure builds up in water at 1 bar/33 feet, so working this out, you have to place the sphere
at the bottom of an ocean 561,000 ft, or 106 miles deep! No ocean that deep on the earth.

However, it's easy to get those pressures within the earth- 17kbar corresponds to a depth of about 56 Km. No problem.

Calculating Poisson’s Ratio from \( K \) and \( \mu \).

\[ \nu = \frac{\lambda}{2(\lambda+\mu)} = 13 \text{ GPa}/2(13 \text{ GPa} + 6 \text{ GPa}) = 13/38 = .34, \text{ which agrees with other estimates.} \]

How Compressible are rocks? Well, let’s make a guess: We know some rocks, the heaviest, have densities
around 3.3 g/cm\(^3\). Now, the mass of the earth is 6 \times 10\(^{24}\) Kg, and its radius is 6371 Km.

so, the average density, \( \rho \), should be the mass of the earth divided by the volume of the earth, or
\[ \rho = \frac{M}{4 \pi r^3} = 5.5 \text{ g/cm}^3 \]

So right off, we can see that the average density of the earth is higher than what we observe in rocks at the surface.
This is due in fact to a large part of the earth being liquid iron, but also to compression of rocks at depth.
How hard are rocks? We can make a guess at it by considering how strong the atomic bonds are which hold rocks together: these are typically of the order of 1 electron-volt, or 1 ev = 10^{-12} ergs. Now, the work done, or energy expended, to compress an object equals the pressure times the change in volume. The typical volume change for a typical atom is about 10^{-24} cm^3, since an atom is typically about an Angstrom, or 10^{-10} meters across, and we can assume that by ripping apart the atom’s binding electrons we change the volume by 100% (a rough guess). So calculating P*ΔV = 1 ev = 10^{-12} ergs, we can then solve for P, which equals 10^{12} dynes/cm^2. Now, one bar of pressure = 10^6 dyne/cm^2, so we need about 1 megabar, or Mbar, to significantly change the volume of a close packed mineral like solid iron.