Homework 2- Gravity and Geoid

1) Black holes are thought to contain densities of the order of $10^{20}$ Kg/meter$^3$. Suppose your orbiting a black hole of diameter 1 Km at a distance of 150 million Km (earth-sun distance) in a spaceship with total mass of 1000 Kg.

a) If we assume that your potential energy is 0 at an infinite distance from the hole, what is your potential energy after you have fallen in towards the black hole from infinitely far away to your orbiting distance? What are the units?

b) What is the potential at that same distance? What are the units?

c) What force does the black hole pull you in with?

d) How many hours does it take you to orbit the black hole?

2) a) What is the difference in the gravitational acceleration between sea level and the summit of Mt. Everest?

b) How much lighter will an 80 Kg climber feel on the summit of Mt Everest than on at sea level? Will this difference matter much in terms of making the mountain easier to climb?

3) a) Remember that the centripetal acceleration that you feel while standing on the equator of the earth is equal mainly to gravity, but there is another force which counteracts gravity slightly, and this is the merry-go-round affect of the earth's rotation trying to 'fling' you off into space. This acceleration is equal to $\omega^2 R$, where $\omega$ is the rotational velocity or the earth, $R$ is the radius of the earth, and the period of rotation (ie, how much time per revolution of the earth) $T = 2\pi/\omega$. What is the rotational velocity of the earth now?

b) If you're standing on the equator, how fast does the earth have to spin in order to make you feel like you're floating, ie, the earth is 'flinging you off' as strongly as it is gravitationally attracting you?,

c) Will this 'fling' factor increase or decrease as you move towards the poles of the earth? Think about being on a merry-go-round.

d) Extra credit. Derive an expression for the centrifugal force you feel flinging you off the earth's surface as a function of your latitude, $\lambda$, and the rotational velocity of the earth, $\omega$.

4) In the following picture, which bottle has greater pressure at the bottom? h is a constant distance, despite my drawing.
b) What is the pressure at the bottom of each, if the density of water is $\rho$ and the density of oil is $0.5\rho$?

5) **Airy Isostasy**: Derive the depths of the roots, $r_1$, $r_2$, and $r_3$, which hold up the following sets of mountains:

6) **Pratt Isostasy**: Derive the relative densities of the following mountains:
7) Explain why the Bouguer anomaly is close to 0 and the free air anomaly is high when mountains are not isostatically compensated.